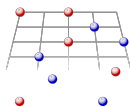


# Topologically non-trivial electronic and magnetic states in doped copper kagome lattices

Daniel Guterding, Harald O. Jeschke, Roser Valentí  
Institut für Theoretische Physik

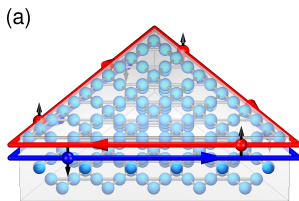
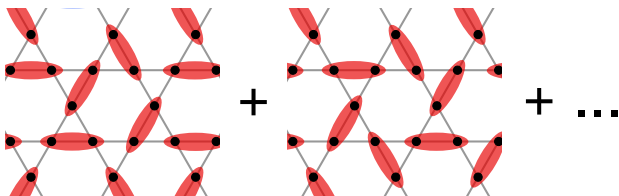
March 18, 2016



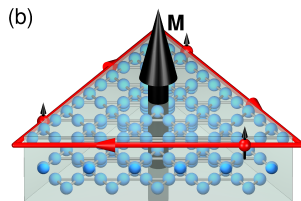
**DFG** Deutsche  
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SFB/TR 49

# Non-trivial topology on the kagome lattice: quantum spin liquid and quantum Hall effects

- highly frustrated lattice
- QSL in the AFM Heisenberg model
- topological electronic edge states (QSHE, QAHE)
- spintronics
- non-abelian anyons (e.g. TI-SC interface)
- topological quantum computer



quantum spin Hall effect

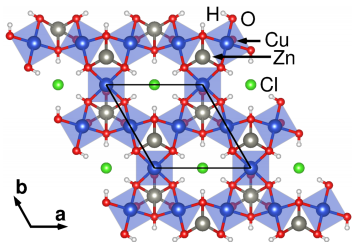


quantum anomalous Hall effect

# Copper kagome materials: Herbertsmithite and Barlowite

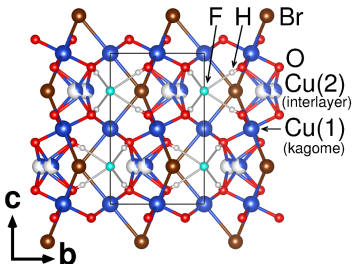
## Herbertsmithite $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

- kagome lattice of Cu  $3d^9$  ( $\text{Cu}^{2+}$ ) ions
  - AFM NN  $J_1 = 182.4$  K, other  $J_i < 7$  K
  - no magnetic order down to 50 mK
- [Helton et al., PRL **98**, 107204 (2007)]
- Cu-Zn antisite-disorder of a few percent



## Barlowite $\text{Cu}_4(\text{OH})_6\text{FBr}$

- $\text{Cu}^{2+}$  between layers, **ordered AFM**
- AFM kagome exchange  $J_3 = 178$  K
- FM interlayer exchange  $J_1 = -205$  K

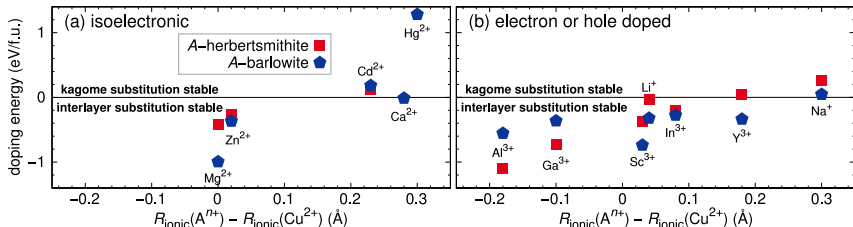
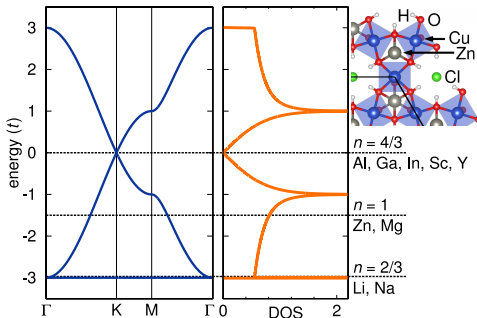


Jeschke, Salvat-Pujol, Valentí, PRB **88**, 075106 (2013)

Jeschke, Salvat-Pujol, Gati, Hoang, Wolf, Lang, Schlueter, Valentí, PRB **92**, 094417 (2015)

# Stability analysis for substituted Herbertsmithite $ACu_3(OH)_6Cl_2$ and Barlowite $ACu_3(OH)_6FBr$

- $A = Li^+, Na^+$ : FM
- $A = Mg^{2+}, Zn^{2+}$ : QSL
- $A = Ga^{3+}, Sc^{3+}$ : Dirac metal
- do dopants occupy interlayer or kagome site? → DFT study
- many materials in principle stable!
- small dopants favored



# Magnetic exchange couplings

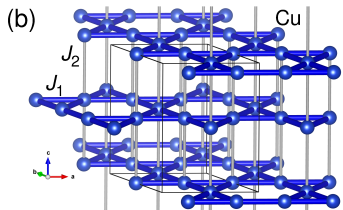
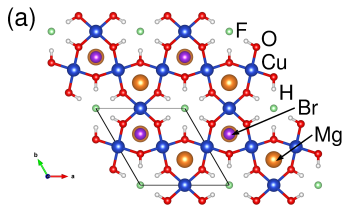
- map DFT+U to Heisenberg model
- $U = 6$  eV and  $J_H = 1$  eV on Cu 3d

## Mg-Barlowite $\text{MgCu}_3(\text{OH})_6\text{FBr}$

- AFM  $J_1 = 226$  K,  $J_2/J_1 = 0.06$
- spin-liquid predicted for  $|J_2/J_1| < 0.15$

[Götze, Richter, arXiv:1510:04898]

- QSL with low anti-site disorder

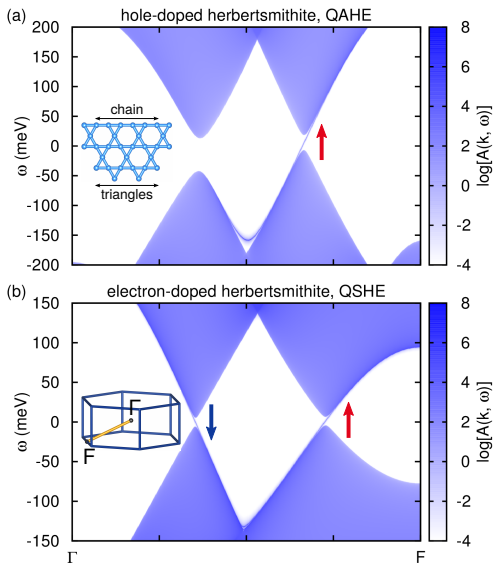


## Li-Herbertsmithite $\text{LiCu}_3(\text{OH})_6\text{Cl}_2$

- FM kagome coupling  $J_1 = -544$  K
- MF Curie temperature  $T_C \approx 1160$  K

# Topological surface states in doped Herbertsmithite

- **Li-Herbertsmithite** is  $2/3$  filled FM,  $\{\uparrow 2/3, \downarrow 0\}$
- **Ga-Herbertsmithite** is  $4/3$  filled,  $\{\uparrow 2/3, \downarrow 2/3\}$
- FM Dirac metal, SOC gap at Dirac point
- $\nu_0; (\nu_1, \nu_2, \nu_3) = 0; (111)$
- **QAHE on (001) surface**
- Barlowite is topologically trivial, additional band crossings



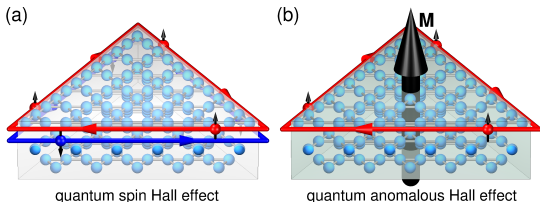
# Doped kagome lattice Mott insulators

## Summary

- investigated stability of new copper kagome materials
- Mg-Barlowite is a new spin-liquid candidate
- Li-Herbertsmithite shows the quantum anomalous Hall effect
- (Al, Ga, In, Sc)-Herbertsmithite show the quantum spin Hall effect
- preprint available: [arXiv:1511.05686](https://arxiv.org/abs/1511.05686) (version 1)

## Outlook

- material synthesis
- gating with ionic liquids
- deposition of alkalis
- intermediate doping

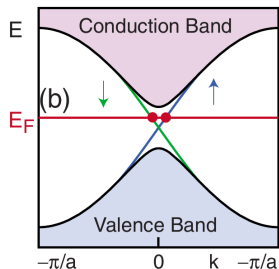
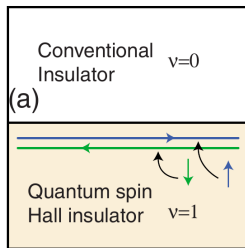
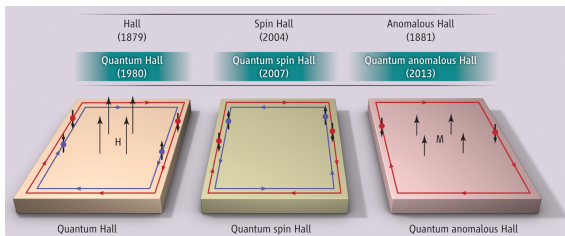


# Appendix



# Topological insulators: introduction

- topological insulator fundamentally different from trivial insulator
- insulating bulk
- **gapless surface states** protected by time-reversal symmetry
- **robust against disorder**, weak interactions, etc.
- dissipation-free transport through surface states



# Topological insulators: computational aspects

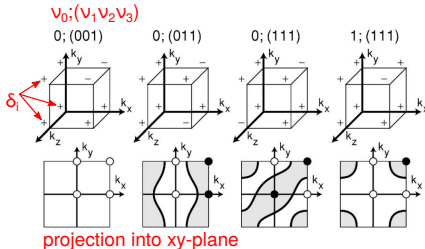
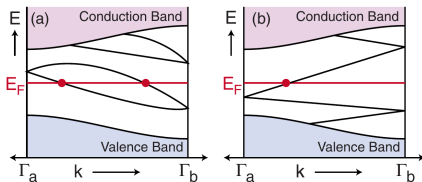
- time-reversal invariance  
 $E(\vec{k}) = E(-\vec{k})$
- Kramer's theorem: time-reversal symmetry  $\rightarrow$  twofold degeneracy
- connectivity between time-reversal invariant points  $\Gamma_i$  matters
- take product of parity eigenvalues for occupied bands

$$\delta_i = \prod_{m=1}^N \xi_{2m}(\Gamma_i)$$

- $(-1)^{\nu_0} = \prod_{i=1}^8 \delta_i; \rightarrow \nu_0; (\nu_1, \nu_2, \nu_3)$

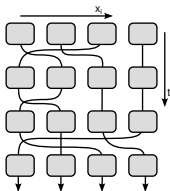
$$(-1)^{\nu_k} =$$

$$\prod_{n_k=1; n_{j \neq k}=0,1} \delta_{i=(n_1 n_2 n_3)}$$

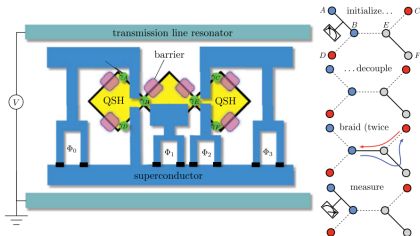
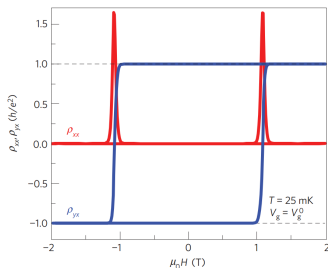


# Topological insulators: application

- in QAH insulator all currents are spin-polarized  $\rightarrow$  spintronics
- $\rho_{xx} = 0$  at zero field for QAHE
- Majorana zero modes (non-abelian anyons) at TI-SC interface
- braiding records history of the system
- fault-tolerant quantum computer



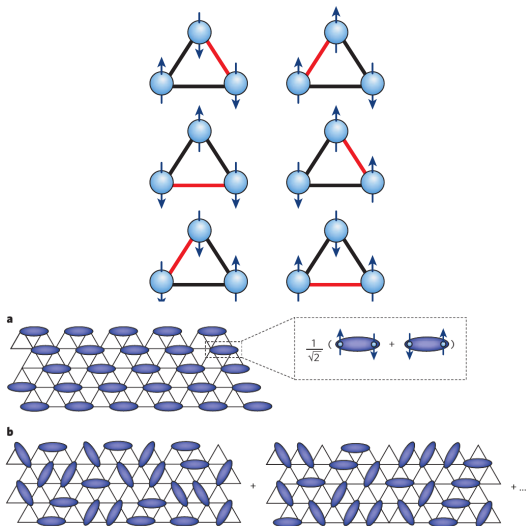
Das Sarma et al., Quantum Information **1**, 15001 (2015); van Heck et al., Phys. Scr. **T164**, 014007 (2015)



Wilczek, Nat. Phys. **5**, 614 (2009); Chang et al., Nat. Mater. **14**, 473 (2015)

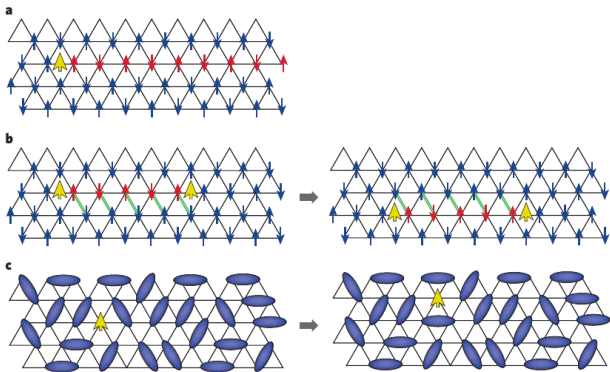
# Quantum spin-liquids: introduction

- frustration leads to degenerate ground state
- highly correlated well-formed local moments, but no static order (liquid-like)
- **no spontaneous symmetry-breaking**
- described by lattice covered with valence bonds
- static valence bonds give valence bond solid (VBS)
- **superposition gives resonating valence bond (RVB) state  $\rightarrow$  QSL**



# Quantum spin-liquids: excitations

- usual excitations are **electron-like** ( $S = 1/2, q = \pm e$ ) or **magnon-like** ( $S = 1, q = 0$ )
- fractionalized excitations in QSL: **spinons** ( $S = 1/2, q = 0$ )
- spinons can be gapped or gapless
- probe spinons with thermodynamics, neutrons, etc.



# Herbertsmithite $[\text{ZnCu}_3(\text{OH})_6\text{Cl}_2]$ : spin-liquid candidate on the kagome lattice

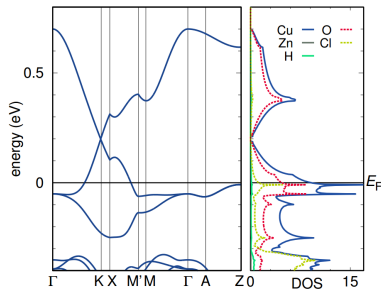
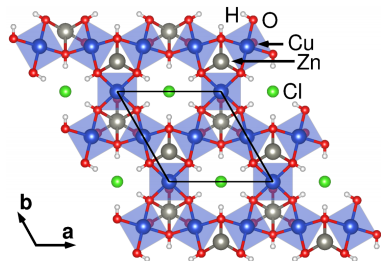
- kagome lattice of Cu  $3d^9$  ( $\text{Cu}^{2+}$ ) ions
- Mott insulator
- antiferromagnetic NN exchange  $J_1 = 182.4$  K
- various other couplings  $J_i < 7$  K
- **no magnetic order down to 50 mK**

[Helton et al., PRL **98**, 107204 (2007)]

- neutron experiments see continuum of (fractionalized) excitations

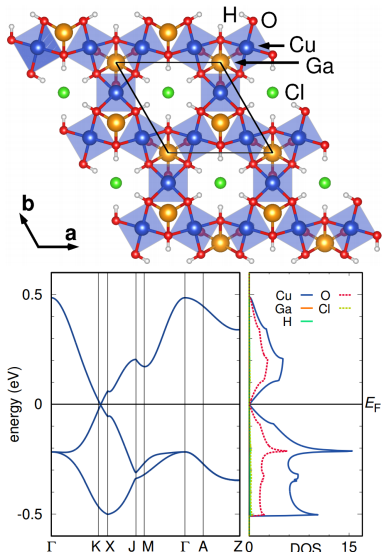
[Han et. al, Nature **492**, 406 (2012)]

- **Cu-Zn antisite-disorder of a few percent**, some studies claim only Zn site disorder



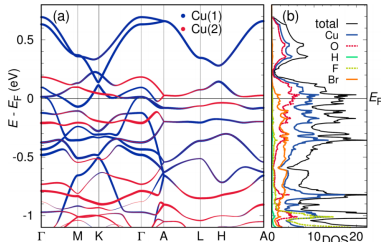
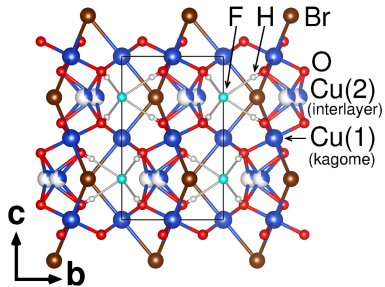
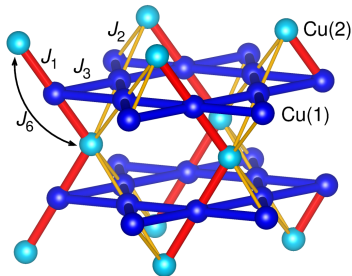
# Ga-Herbertsmithite $[\text{GaCu}_3(\text{OH})_6\text{Cl}_2]$ : predicted exotic ferromagnet or superconductor

- substitute  $\text{Zn}^{2+}$  by  $\text{Ga}^{3+}$
- electron doping moves Fermi level to Dirac point
- Dirac metal robust in presence of correlations (DCA)
- vary Zn:Ga ratio to obtain intermediate electron doping
- competition between FM and f-wave SC predicted



# Barlowite $[\text{Cu}_4(\text{OH})_6\text{FBr}]$ : kagome lattice antiferromagnet

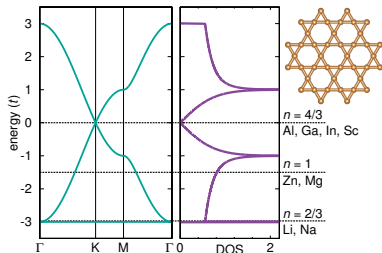
- kagome lattice of  $\text{Cu}^{2+}$  ions
- $\text{Cu}^{2+}$  also between kagome layers
- **antiferromagnetic Mott insulator**
- AFM kagome exchange  $J_3 = 178$  K
- FM interlayer exchange  $J_1 = -205$  K
- remove interlayer coupling for QSL



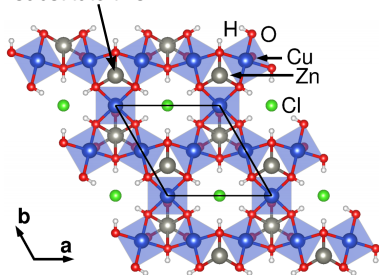


# Chemical modifications of Barlowite and Herbertsmithite

- we investigate  
A-Herbertsmithite [ $ACu_3(OH)_6Cl_2$ ] and  
A-Barlowite [ $ACu_3(OH)_6FBr$ ]
- we propose dopants for realizing QAHE, QSHE and QSL
- hole-doping (QAHE):  $Li^+$ ,  $Na^+$
- electron-doping (QSHE):  $Al^{3+}$ ,  $Ga^{3+}$ ,  $In^{3+}$ ,  $Sc^{3+}$ ,  $Y^{3+}$
- isoelectronic doping (QSL):  $Mg^{2+}$ ,  $Ca^{2+}$ ,  $Zn^{2+}$ ,  $Cd^{2+}$ ,  $Hg^{2+}$
- for all proposed compounds we analyse the stability of crystal structures
- Is kagome lattice distorted upon doping?



substitute this



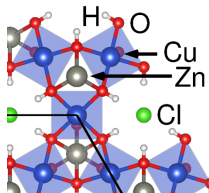
# Preparation and analysis of crystal structures

## structure preparation and relaxation

- replace interlayer site starting from Herbertsmithite/Barlowite
- density functional theory (DFT) calculations, GGA-PBE functional
- GPAW code, projector-augmented wave (PAW) basis, 1000 eV plane-wave cutoff
- **unit cell shape and internal coordinates relaxed** until forces  $< 10 \text{ meV}/\text{\AA}$

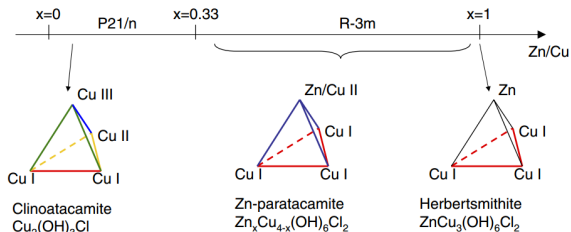
## stability tests for $\text{ACu}_3(\text{OH})_6\text{Cl}_2$ and $\text{ACu}_3(\text{OH})_6\text{FBr}$

- 1 does the dopant go in at all?
- 2 **does the dopant like the kagome environment better?**
- 3 what about vacancies instead of A?
- 4 what about Cu impurities instead of A?



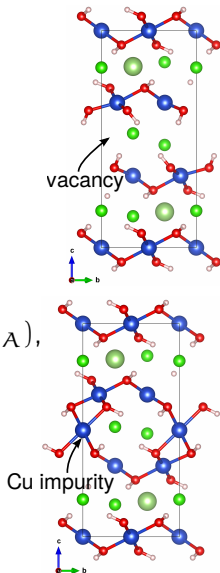
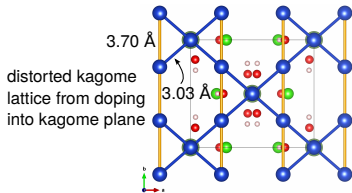
# Formation energies

- total energies from full-potential local orbital (FPLO) code
- model growth process by solid state reaction
- compare energies of  $ACu_3(OH)_6Cl_2 + Cu$  to  $2 \times Cu_2(OH)_3Cl + A$
- dopant metal energies from crystals, e.g. fcc-Cu
- formation energy is defined as  $E_{form} = E_{AH} + E_{Cu} - (E_C + E_A)$
- negative formation energy  $\rightarrow ACu_3(OH)_6Cl_2$  is formed
- positive formation energy  $\rightarrow Cu_2(OH)_3Cl$  is formed



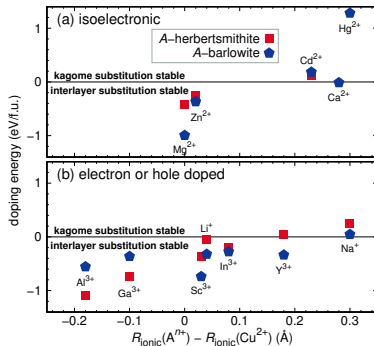
# Doping Energies

- evaluate energies of various defects, fully relaxed structures
- dopant A on kagome site and Cu on interlayer  
 $\text{Cu}(\text{ACu}_2)(\text{OH})_6\text{Cl}_2$
- compare energy directly to  $\text{ACu}_3(\text{OH})_6\text{Cl}_2$
- vacancy structure  $\text{A}_{0.66}\text{Cu}_3(\text{OH})_6\text{Cl}_2$
- impurity structure  $\text{A}_{0.66}\text{Cu}_{3.33}(\text{OH})_6\text{Cl}_2$
- doping energy:  $E_{\text{dop}} = E_{\text{AH}} + y \cdot E_{\text{Cu}} - (E_{\text{AHM}} + x \cdot E_{\text{A}})$ ,  
where  $E_{\text{AHM}}$  belongs to  $\text{A}_{1-x}\text{Cu}_{3+y}(\text{OH})_6\text{Cl}_2$



# Summary of formation and doping energies

- variety of materials energetically possible
- vacancies and large fractions of Cu impurities irrelevant
- interlayer site favors small dopants
- hole-doped Li-Herbertsmithite stable
- various electron doped materials stable
- Mg-Barlowite strongly favors interlayer doping, probably reduced disorder

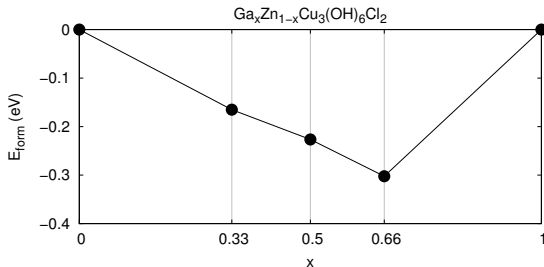


case	A-herbertsmithite, A =	Li <sup>1+</sup>	Na <sup>1+</sup>	Mg <sup>2+</sup>	Ca <sup>2+</sup>	Zn <sup>2+</sup>	Cd <sup>2+</sup>	Hg <sup>2+</sup>	Sc <sup>3+</sup>	Y <sup>3+</sup>	Al <sup>3+</sup>	Ga <sup>3+</sup>	In <sup>3+</sup>
1	ACu <sub>3</sub> (OH) <sub>6</sub> Cl <sub>2</sub>	-2.660	-2.517	-5.333	n/a	-2.082	-1.461	n/a	-7.300	-7.144	-6.257	-2.939	-3.080
2	ACu <sub>3</sub> (OH) <sub>6</sub> Cl <sub>2</sub>	-0.041	+0.257	-0.421	n/a	-0.261	+0.128	n/a	-0.372	+0.043	-1.101	-0.736	-0.203
3	A <sub>0.66</sub> Cu <sub>3</sub> (OH) <sub>6</sub> Cl <sub>2</sub>	-0.847	-0.735	-1.689	n/a	-0.694	n/a	n/a	-2.481	-2.398	-2.141	-1.176	-1.062
4	A <sub>0.66</sub> Cu <sub>3.33</sub> (OH) <sub>6</sub> Cl <sub>2</sub>	-1.497	-1.350	-2.308	n/a	-1.321	n/a	n/a	-3.019	-2.878	-2.659	-1.711	-1.585
	A-barlowite, A =												
1	ACu <sub>3</sub> (OH) <sub>6</sub> FBr	-2.860	-2.247	-5.089	-5.808	-2.074	-1.420	+1.317	-7.498	-7.400	-5.605	-2.895	-2.919
2	ACu <sub>3</sub> (OH) <sub>6</sub> FBr	-0.322	+0.047	-0.994	-0.012	-0.367	+0.183	+1.282	-0.738	-0.336	-0.555	-0.363	-0.276
	ionic radius in pm	76	102	72	100	74	95	102	75	90	54	62	80

# Doping energies for small concentrations

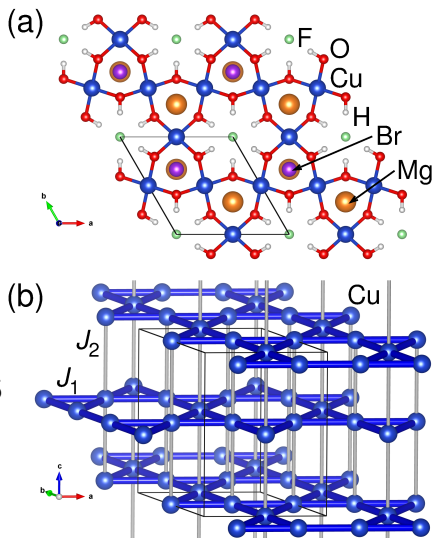
- Herbertsmithite very stable against excess copper on interlayer site
- Herbertsmithite susceptible to excess dopants in kagome plane
- more work needed to understand impurity concentration, arrangement, etc.
- solid solution  $\text{Ga}_x\text{Zn}_{1-x}\text{Cu}_3(\text{OH})_6\text{Cl}_2$  should exist

$A =$	$\text{Zn}^{2+}$	$\text{Mg}^{2+}$
$A_{0.83}\text{Cu}_{3.17}(\text{OH})_6\text{Cl}_2$	-0.464	-0.831
$A_{1.17}\text{Cu}_{2.83}(\text{OH})_6\text{Cl}_2$	+0.210	+0.803



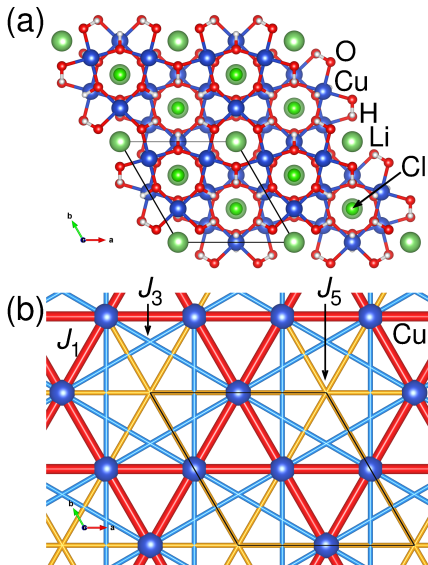
# Heisenberg model for Mg-Barlowite

- DFT+U calculation with  $U = 6$  eV and  $J_H = 1$  eV on Cu 3d
- map energies of DFT spin-configurations to Heisenberg model
- **AFM kagome coupling  $J_1 = 226$  K**
- interlayer coupling  $J_2 = 13.4$  K
- $J_2/J_1 = 0.06$
- spin-liquid predicted for  $|J_2/J_1| < 0.15$   
[Götze, Richter, arXiv:1510:04898]
- **Mg-Barlowite is likely a quantum spin-liquid with low anti-site disorder**



# Heisenberg model for Li-Herbertsmithite

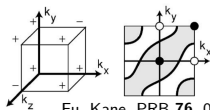
- DFT+U calculation with  $U = 6$  eV and  $J_H = 1$  eV on Cu 3d
- large FM kagome coupling  
 $J_1 = -544$  K
- negligible interplane couplings
- mean-field Curie temperature  
 $T_C = -\frac{2}{3}S(S+1) \sum_i z_i J_i$
- $J_i = \{-544, 39, -50\}$  K and  $z_i = \{4, 4, 6\} \rightarrow T_C \approx 1160$  K
- Li-Herbertsmithite is a very robust ferromagnet



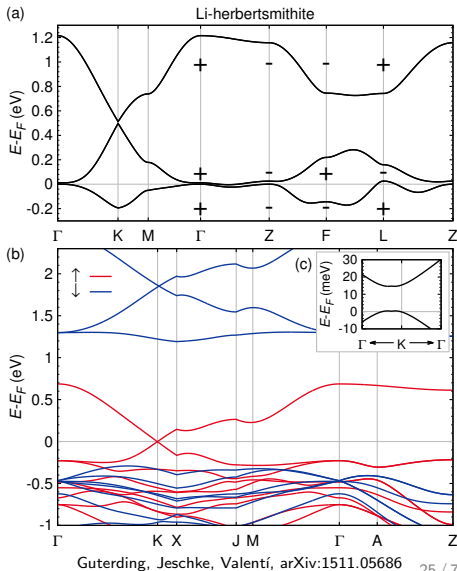


# Electronic properties of Li-Herbertsmithite

- Ga-Herbertsmithite is 4/3 filled,  $\{\uparrow 2/3, \downarrow 2/3\}$
- Li-Herbertsmithite is 2/3 filled FM,  $\{\uparrow 2/3, \downarrow 0\}$
- FM Dirac metal is suspicious of QAHE
- SOC opens gap at Dirac point
- check topological indices, parity eigenvalues from DFT
- $\nu_0; (\nu_1, \nu_2, \nu_3) = 0; (111)$
- QAHE on (001) surface

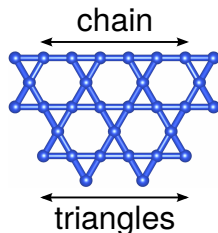
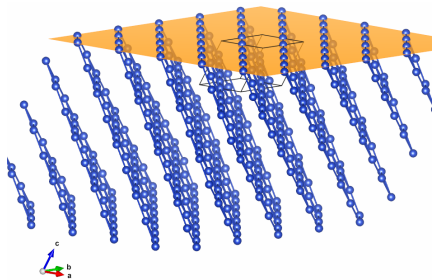


Fu, Kane, PRB **76**, 045302 (2007)



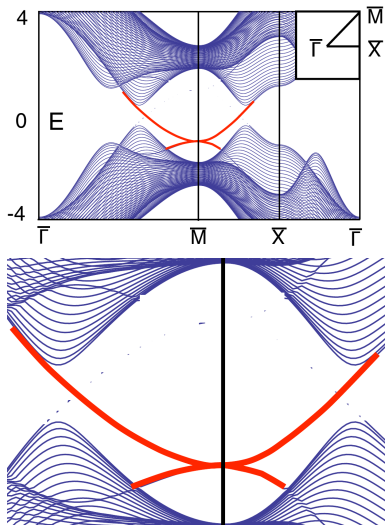
# Surface termination of Herbertsmithite

- $\nu_0; (\nu_1, \nu_2, \nu_3) = 0; (111)$  is stack of 2D systems
- (001) termination has edge of kagome layer at surface
- termination influences details of surface bands, not qualitative physics
- **surface states are generic in Herbertsmithite system**
- doping only controls Fermi level
- **electron doped-materials show QSHE**, two spins at Fermi level
- Barlowite bands are topologically trivial, additional band crossings

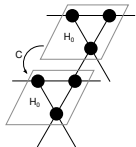


# How to calculate surface states

- full-relativistic DFT calculations, projective Wannier functions
- tight binding model for Cu states  $(n, j, m_j) = (3, 5/2, \pm 5/2)$
- **minimum number of layers to allow bulk-surface distinction  $N \sim 20$**
- diagonalization effort grows with  $N^3$
- **always two surfaces in TB slab**, experiment observes one at a time
- some authors just erase bands from the dual surface (justified, but ugly)
- proper way to calculate surface states: Green's functions



# State on the surface of Herbertsmithite



- iterative Green's function method

[Sancho et al., J. Phys. F: Met. Phys. **14**, 1205 (1984)]

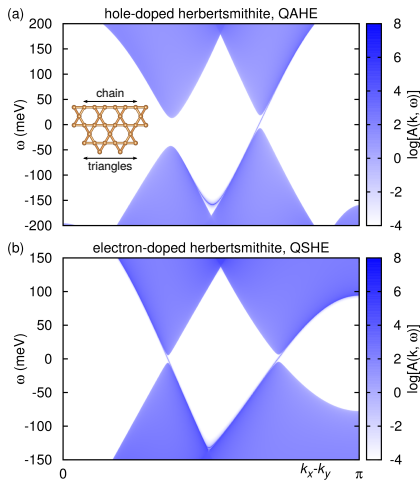
- initial condition  $G_{ij}^{(1)}(\omega) = (\omega - H_0)_{ij}^{-1}$

- surface  $G_{ij}^{(N)}(\omega) = \left[ (\omega - H_0 - CG^{(N-1)}C^\dagger)^{-1} \right]_{ij}$

- dual surface  $G_{ij}^{(N)}(\omega) = \left[ (\omega - H_0 - C^\dagger G^{(N-1)}C)^{-1} \right]_{ij}$

- spectral function

$$A(\mathbf{k}, \omega) = -\text{Im Tr} [G(\mathbf{k}, \omega)] / \pi$$

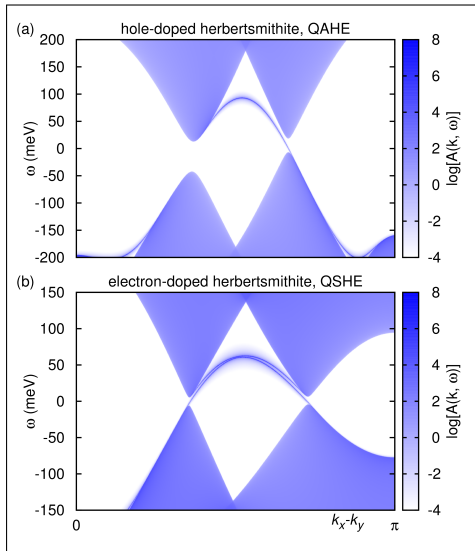
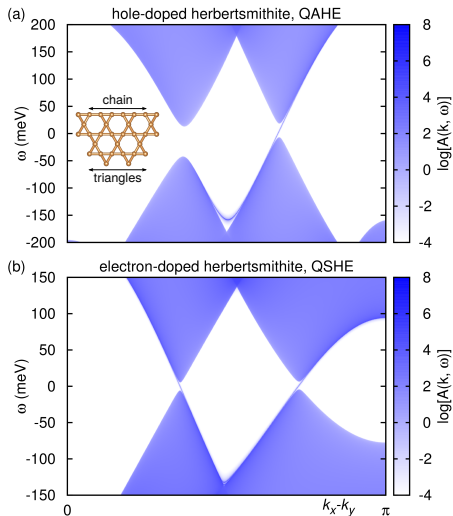


Guterding, Jeschke, Valentí, arXiv:1511.05686

- $N = 10^5$

- $\omega \rightarrow \omega + i \cdot 10^{-5} \text{ eV}$

# State on the dual surface of Herbertsmithite



# Test for Green's function code: crystalline topological insulator

