

Topologically non-trivial electronic and magnetic states in doped copper Kagome lattices

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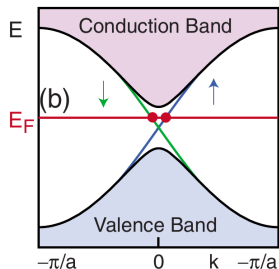
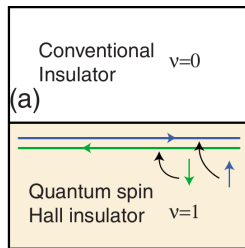
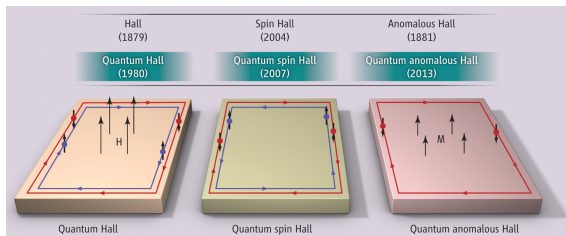
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Outline

- 1 Introduction to Quantum Spin Hall and Quantum Anomalous Hall effect
- 2 Introduction to Spin-Liquids
- 3 Current status of copper Kagome materials
- 4 Suggestions for chemical modifications
- 5 Stability analysis for proposed crystal structures
- 6 Parametrization of Heisenberg Hamiltonians
- 7 Spin-Liquid in Barlowite
- 8 Quantum Spin Hall effect and Quantum Anomalous Hall effect in Herbertsmithite

Topological insulators: introduction

- topological insulator fundamentally different from trivial insulator
- insulating bulk
- **gapless surface states** protected by time-reversal symmetry
- **robust against disorder**, weak interactions, etc.
- dissipation-free transport through surface states

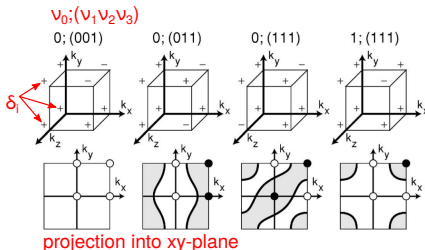
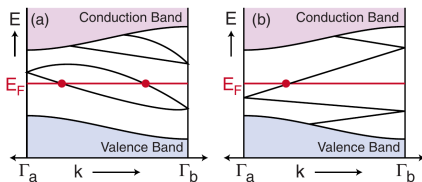


Topological insulators: computational aspects

- time-reversal invariance
 $E(\vec{k}) = E(-\vec{k})$
- Kramer's theorem: time-reversal symmetry \rightarrow twofold degeneracy
- connectivity between time-reversal invariant points Γ_i matters
- take product of parity eigenvalues for occupied bands

$$\delta_i = \prod_{m=1}^N \xi_{2m}(\Gamma_i)$$

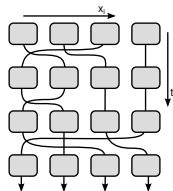
- $(-1)^{\nu_0} = \prod_{i=1}^8 \delta_i; \rightarrow \nu_0; (\nu_1, \nu_2, \nu_3)$
- $(-1)^{\nu_k} = \prod_{n_k=1; n_{j \neq k}=0,1} \delta_{i=(n_1 n_2 n_3)}$



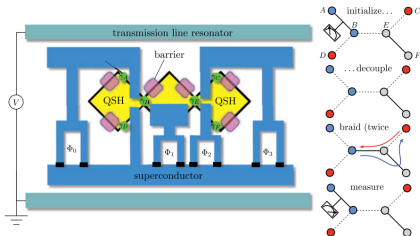
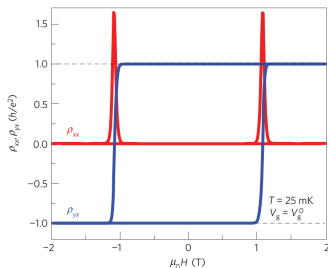
projection into xy-plane

Topological insulators: application

- in QAH insulator all currents are spin-polarized → spintronics
- $\rho_{xx} = 0$ at zero field for QAHE
- Majorana zero modes (non-abelian anyons) at TI-SC interface
- braiding records history of the system
- fault-tolerant quantum computer



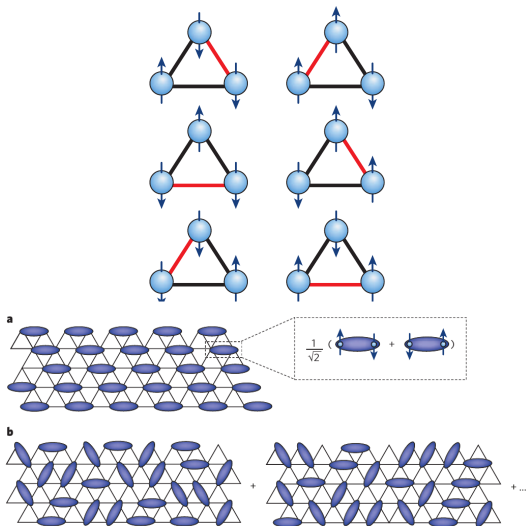
Das Sarma et al., Quantum Information **1**, 15001 (2015); van Heck et al., Phys. Scr. **T164**, 014007 (2015)



Wilczek, Nat. Phys. **5**, 614 (2009); Chang et al., Nat. Mater. **14**, 473 (2015)

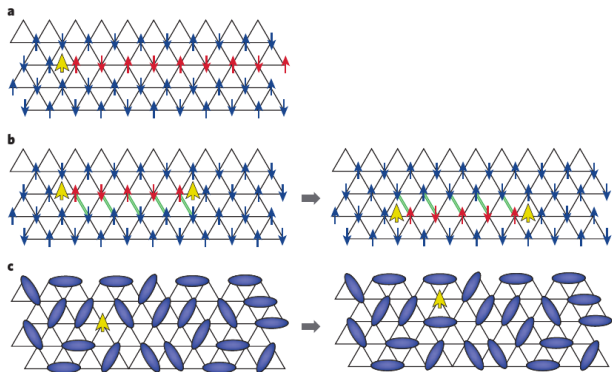
Quantum spin-liquids: introduction

- frustration leads to degenerate ground state
- highly correlated well-formed local moments, but no static order (liquid-like)
- **no spontaneous symmetry-breaking**
- described by lattice covered with valence bonds
- static valence bonds give valence bond solid (VBS)
- **superposition gives resonating valence bond (RVB) state \rightarrow QSL**



Quantum spin-liquids: excitations

- usual excitations are **electron-like** ($S = 1/2, q = \pm e$) or **magnon-like** ($S = 1, q = 0$)
- fractionalized excitations in QSL: **spinons** ($S = 1/2, q = 0$)
- spinons can be gapped or gapless
- probe spinons with thermodynamics, neutrons, etc.



Herbertsmithite $[\text{ZnCu}_3(\text{OH})_6\text{Cl}_2]$: spin-liquid candidate on the kagome lattice

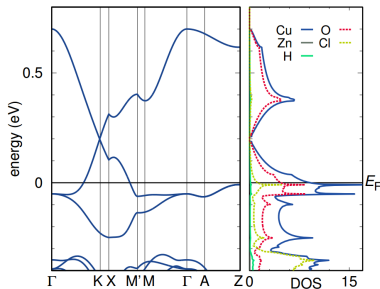
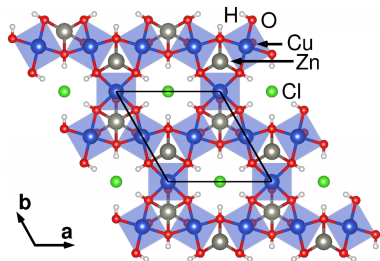
- kagome lattice of Cu $3d^9$ (Cu^{2+}) ions
- Mott insulator
- antiferromagnetic NN exchange $J_1 = 182.4$ K
- various other couplings $J_i < 7$ K
- **no magnetic order down to 50 mK**

[Helton et al., PRL **98**, 107204 (2007)]

- neutron experiments see continuum of (fractionalized) excitations

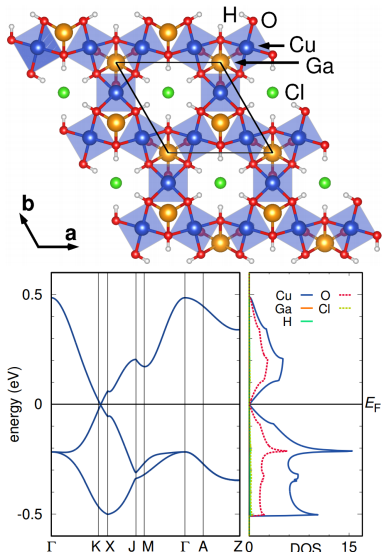
[Han et. al, Nature **492**, 406 (2012)]

- **Cu-Zn antisite-disorder of a few percent**, some studies claim only Zn site disorder



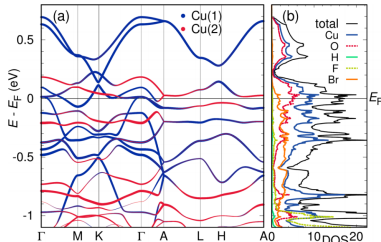
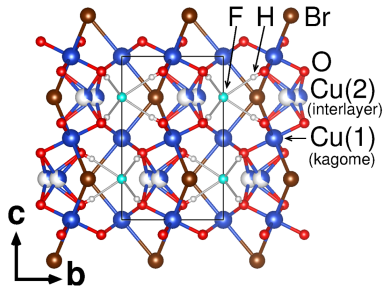
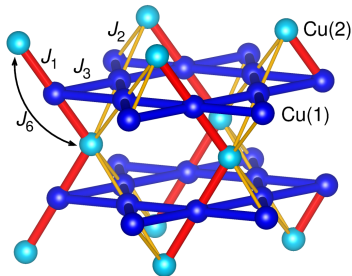
Ga-Herbertsmithite $[\text{GaCu}_3(\text{OH})_6\text{Cl}_2]$: predicted exotic ferromagnet or superconductor

- substitute Zn^{2+} by Ga^{3+}
- electron doping moves Fermi level to Dirac point
- Dirac metal robust in presence of correlations (DCA)
- vary Zn:Ga ratio to obtain intermediate electron doping
- competition between FM and f-wave SC predicted



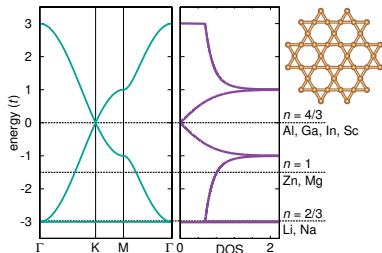
Barlowite $[\text{Cu}_4(\text{OH})_6\text{FBr}]$: kagome lattice antiferromagnet

- kagome lattice of Cu^{2+} ions
- Cu^{2+} also between kagome layers
- **antiferromagnetic Mott insulator**
- AFM kagome exchange $J_3 = 178 \text{ K}$
- FM interlayer exchange $J_1 = -205 \text{ K}$
- remove interlayer coupling for QSL

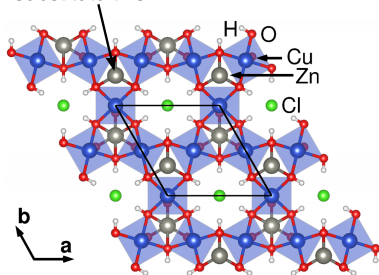


Chemical modifications of Barlowite and Herbertsmithite

- we investigate
A-Herbertsmithite [$ACu_3(OH)_6Cl_2$] and
A-Barlowite [$ACu_3(OH)_6FBr$]
- we propose dopants for realizing QAHE, QSHE and QSL
- hole-doping (QAHE): Li^+ , Na^+
- electron-doping (QSHE): Al^{3+} , Ga^{3+} , In^{3+} , Sc^{3+} , Y^{3+}
- isoelectronic doping (QSL): Mg^{2+} , Ca^{2+} , Zn^{2+} , Cd^{2+} , Hg^{2+}
- for all proposed compounds we analyse the stability of crystal structures
- Is kagome lattice distorted upon doping?



substitute this



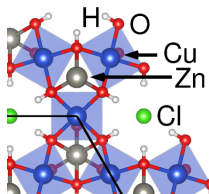
Preparation and analysis of crystal structures

structure preparation and relaxation

- replace interlayer site starting from Herbertsmithite/Barlowite
- density functional theory (DFT) calculations, GGA-PBE functional
- GPAW code, projector-augmented wave (PAW) basis, 1000 eV plane-wave cutoff
- **unit cell shape and internal coordinates relaxed** until forces < 10 meV/Å

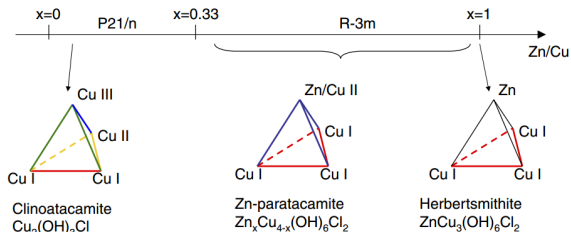
stability tests for $ACu_3(OH)_6Cl_2$ and $ACu_3(OH)_6FBr$

- 1 does the dopant go in at all?
- 2 **does the dopant like the kagome environment better?**
- 3 what about vacancies instead of A?
- 4 what about Cu impurities instead of A?



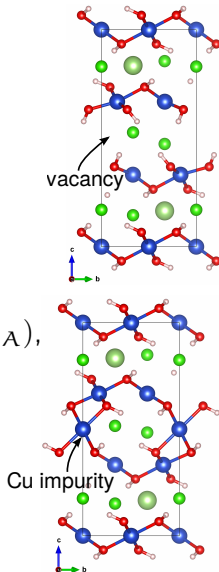
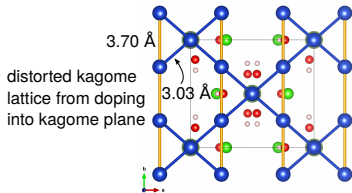
Formation energies

- total energies from full-potential local orbital (FPLO) code
- model growth process by solid state reaction
- compare energies of $ACu_3(OH)_6Cl_2 + Cu$ to $2 \times Cu_2(OH)_3Cl + A$
- dopant metal energies from crystals, e.g. fcc-Cu
- formation energy is defined as $E_{form} = E_{AH} + E_{Cu} - (E_C + E_A)$
- negative formation energy $\rightarrow ACu_3(OH)_6Cl_2$ is formed
- positive formation energy $\rightarrow Cu_2(OH)_3Cl$ is formed



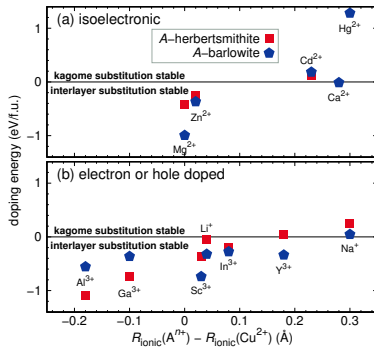
Doping Energies

- evaluate energies of various defects, fully relaxed structures
- dopant A on kagome site and Cu on interlayer
 $\text{Cu}(\text{ACu}_2)(\text{OH})_6\text{Cl}_2$
- compare energy directly to $\text{ACu}_3(\text{OH})_6\text{Cl}_2$
- vacancy structure $\text{A}_{0.66}\text{Cu}_3(\text{OH})_6\text{Cl}_2$
- impurity structure $\text{A}_{0.66}\text{Cu}_{3.33}(\text{OH})_6\text{Cl}_2$
- doping energy: $E_{\text{dop}} = E_{\text{AH}} + y \cdot E_{\text{Cu}} - (E_{\text{AHM}} + x \cdot E_{\text{A}})$,
 where E_{AHM} belongs to $\text{A}_{1-x}\text{Cu}_{3+y}(\text{OH})_6\text{Cl}_2$



Summary of formation and doping energies

- variety of materials energetically possible
- vacancies and large fractions of Cu impurities irrelevant
- interlayer site favors small dopants
- hole-doped Li-Herbertsmithite stable
- various electron doped materials stable
- Mg-Barlowite strongly favors interlayer doping, probably reduced disorder

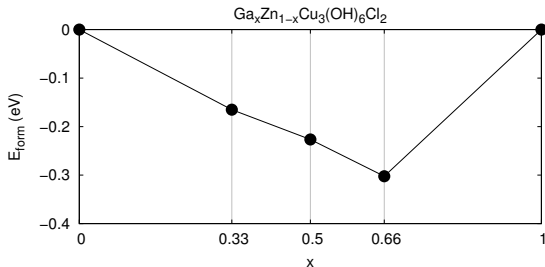


case	A-herbertsmithite, A =	Li ¹⁺	Na ¹⁺	Mg ²⁺	Ca ²⁺	Zn ²⁺	Cd ²⁺	Hg ²⁺	Sc ³⁺	Y ³⁺	Al ³⁺	Ga ³⁺	In ³⁺
1	ACu ₃ (OH) ₆ Cl ₂	-2.660	-2.517	-5.333	n/a	-2.082	-1.461	n/a	-7.300	-7.144	-6.257	-2.939	-3.080
2	ACu ₃ (OH) ₆ Cl ₂	-0.041	+0.257	-0.421	n/a	-0.261	+0.128	n/a	-0.372	+0.043	-1.101	-0.736	-0.203
3	A _{0.66} Cu ₃ (OH) ₆ Cl ₂	-0.847	-0.735	-1.689	n/a	-0.694	n/a	n/a	-2.481	-2.398	-2.141	-1.176	-1.062
4	A _{0.66} Cu _{3.33} (OH) ₆ Cl ₂	-1.497	-1.350	-2.308	n/a	-1.321	n/a	n/a	-3.019	-2.878	-2.659	-1.711	-1.585
	A-barlowite, A =												
1	ACu ₃ (OH) ₆ FBr	-2.860	-2.247	-5.089	-5.808	-2.074	-1.420	+1.317	-7.498	-7.400	-5.605	-2.895	-2.919
2	ACu ₃ (OH) ₆ FBr	-0.322	+0.047	-0.994	-0.012	-0.367	+0.183	+1.282	-0.738	-0.336	-0.555	-0.363	-0.276
	ionic radius in pm	76	102	72	100	74	95	102	75	90	54	62	80

Doping energies for small concentrations

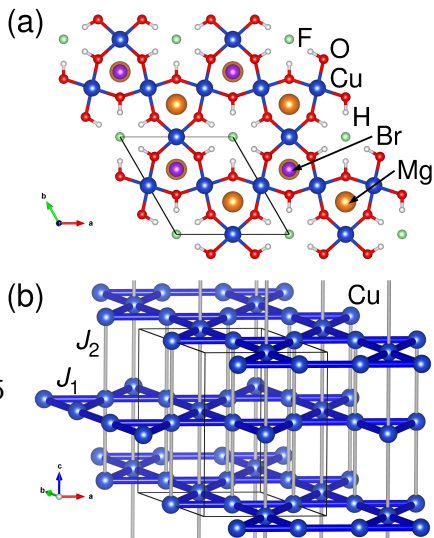
- Herbertsmithite very stable against excess copper on interlayer site
- Herbertsmithite susceptible to excess dopants in kagome plane
- more work needed to understand impurity concentration, arrangement, etc.
- solid solution $\text{Ga}_x\text{Zn}_{1-x}\text{Cu}_3(\text{OH})_6\text{Cl}_2$ should exist

$A =$	Zn^{2+}	Mg^{2+}
$A_{0.83}\text{Cu}_{3.17}(\text{OH})_6\text{Cl}_2$	-0.464	-0.831
$A_{1.17}\text{Cu}_{2.83}(\text{OH})_6\text{Cl}_2$	+0.210	+0.803



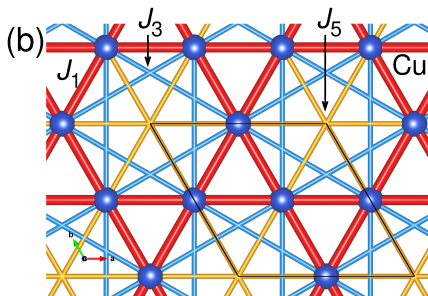
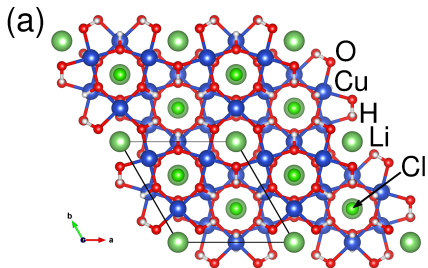
Heisenberg model for Mg-Barlowite

- DFT+U calculation with $U = 6$ eV and $J_H = 1$ eV on Cu 3d
- map energies of DFT spin-configurations to Heisenberg model
- **AFM kagome coupling $J_1 = 226$ K**
- interlayer coupling $J_2 = 13.4$ K
- $J_2/J_1 = 0.06$
- spin-liquid predicted for $|J_2/J_1| < 0.15$
[Götze, Richter, arXiv:1510:04898]
- **Mg-Barlowite is likely a quantum spin-liquid with low anti-site disorder**



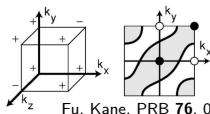
Heisenberg model for Li-Herbertsmithite

- DFT+U calculation with $U = 6$ eV and $J_H = 1$ eV on Cu 3d
- large FM kagome coupling
 $J_1 = -544$ K
- negligible interplane couplings
- mean-field Curie temperature
 $T_C = -\frac{2}{3}S(S+1) \sum_i z_i J_i$
- $J_i = \{-544, 39, -50\}$ K and $z_i = \{4, 4, 6\} \rightarrow T_C \approx 1160$ K
- Li-Herbertsmithite is a very robust ferromagnet

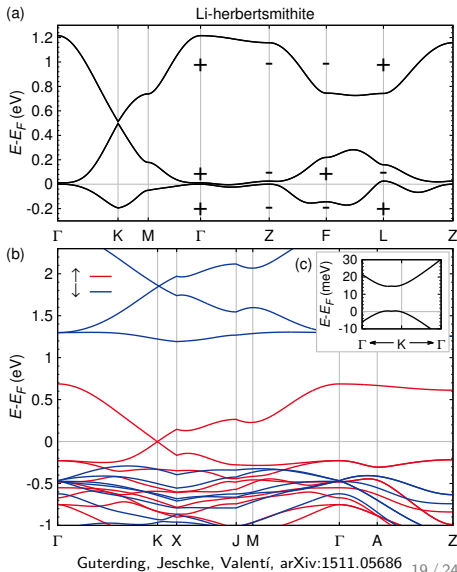


Electronic properties of Li-Herbertsmithite

- Ga-Herbertsmithite is 4/3 filled, $\{\uparrow 2/3, \downarrow 2/3\}$
- Li-Herbertsmithite is 2/3 filled FM, $\{\uparrow 2/3, \downarrow 0\}$
- FM Dirac metal is suspicious of QAHE
- SOC opens gap at Dirac point
- check topological indices, parity eigenvalues from DFT
- $\nu_0; (\nu_1, \nu_2, \nu_3) = 0; (111)$
- QAHE on (001) surface

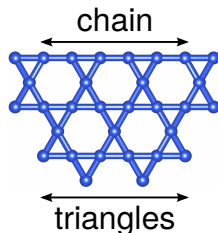
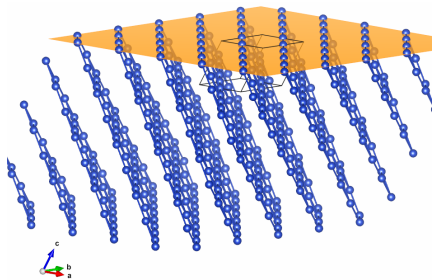


Fu, Kane, PRB **76**, 045302 (2007)



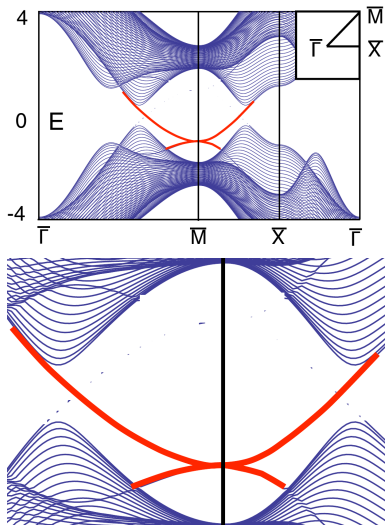
Surface termination of Herbertsmithite

- $\nu_0; (\nu_1, \nu_2, \nu_3) = 0; (111)$ is stack of 2D systems
- (001) termination has edge of kagome layer at surface
- termination influences details of surface bands, not qualitative physics
- surface states are generic in Herbertsmithite system
- doping only controls Fermi level
- electron doped-materials show QSHE, two spins at Fermi level
- Barlowite bands are topologically trivial, additional band crossings

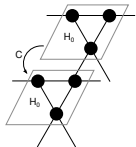


How to calculate surface states

- full-relativistic DFT calculations, projective Wannier functions
- tight binding model for Cu states $(n, j, m_j) = (3, 5/2, \pm 5/2)$
- **minimum number of layers to allow bulk-surface distinction $N \sim 20$**
- diagonalization effort grows with N^3
- **always two surfaces in TB slab**, experiment observes one at a time
- some authors just erase bands from the dual surface (justified, but ugly)
- proper way to calculate surface states: Green's functions



State on the surface of Herbertsmithite



- iterative Green's function method

[Sancho et al., J. Phys. F: Met. Phys. **14**, 1205 (1984)]

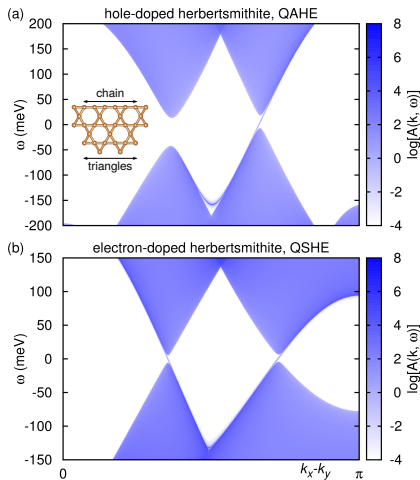
- initial condition $G_{ij}^{(1)}(\omega) = (\omega - H_0)_{ij}^{-1}$

- surface $G_{ij}^{(N)}(\omega) = \left[(\omega - H_0 - CG^{(N-1)}C^\dagger)^{-1} \right]_{ij}$

- dual surface $G_{ij}^{(N)}(\omega) = \left[(\omega - H_0 - C^\dagger G^{(N-1)}C)^{-1} \right]_{ij}$

- spectral function

$$A(k, \omega) = -\text{Im Tr} [G(k, \omega)] / \pi$$

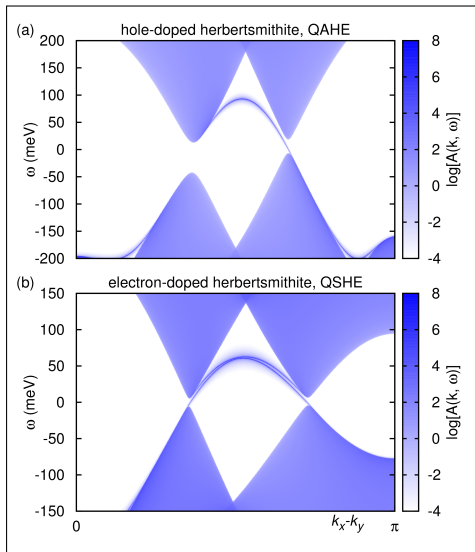
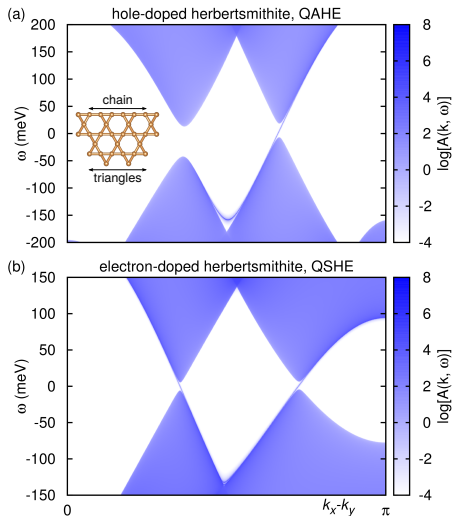


Guterding, Jeschke, Valentí, arXiv:1511.05686

- $N = 10^5$

- $\omega \rightarrow \omega + i \cdot 10^{-5} \text{ eV}$

State on the dual surface of Herbertsmithite



Summary and outlook

Summary

- investigated stability of a range of new copper kagome materials
- all materials feature strong electronic correlations
- substitution of interlayer ions offers tunability
- Mg-Barlowite is a new spin-liquid candidate
- Li-Herbertsmithite shows the quantum anomalous Hall effect
- (Al, Ga, In, Sc)-Herbertsmithite show the quantum spin Hall effect
- preprint available: [arXiv:1511.05686](https://arxiv.org/abs/1511.05686)

Outlook

- correlation effects beyond DFT
- material synthesis

Backup slides

Test for Green's function code: crystalline topological insulator

